3.2.1. Framework

- **Cash Flows.** Seabury ERM employs the well established cash flow methodology of RiskMetrics®. This methodology far exceeds the "cash flow testing" standards specified by Actuarial Standards of Practice No. 7 [16]. While RiskMetrics' framework is focused on short-term (3–10 days) trading VaR, we employ a much longer horizon of one year.
- **Financial Risk Factors.** Each asset and liability may have its own idiosyncratic risk, yet be affected by macroeconomic factors. Seabury ERM is a multi-factor model that employs such risk factors as interest rates of different maturity, equity sector returns, and foreign exchange rates in order to capture the effect of the macroeconomic environment.
- **Risk Categories.** Bottom-up approach allows for the analysis of the various aspects of the company risk. ERM emphasizes the following categories:
    - Credit Risk
    - Interest Rate Risk
    - Insurance Risk
    - Equity Risk
    - Currency Risk
    - Catastrophe Risk

- **Invested Assets.** ERM covers all instruments that are held by investment portfolios such as government, municipal and corporate bonds, asset-backed and mortgage-backed securities (ABS/MBS), common and preferred stocks. All positions are handled at a CUSIP level with the most accurate full valuation algorithms applied.
- **Insurance Liabilities.** Seabury ERM employs an extensive database of insurance losses of all US insurance companies and is working to acquire insurance loss data for the other markets. Seabury ERM is based and improves upon the "stochastic reserving" models put forward by Zehnwirth [14], England [15], and others. The incremental accident year losses are subject to the trends due to varying exposure (accident-year trend), development (development-year trend), and inflation (payment-year trend). ERM jointly models the development patterns and development-year dependent loss volatility. Catastrophe risk and credit risk embedded in reinsurance receivables are also explicitly modeled within the integrated framework.
- **Simulation.** Seabury ERM utilizes a Quasi Monte Carlo technique [17]supplemented by additional regular Monte Carlo randomization. Due to the use of high performance Quasi Monte Carlo methods based on Korobov's lattice rules [18], ERM achieves the speed and rate of convergence impossible with regular Monte Carlo methods (Appendix A:).

3.2.1.1. Cash Flow Methodology

A portfolio of financial instruments may be broken down into a number of future cash flows associated with each position. However, in the VaR calculation, the large number of combinations of possible cash flow dates leads to the impractical task of computing an intractable number of volatilities and correlations. The RiskMetrics methodology [5] drastically simplifies the time structure by mapping each cash flow to a pre-specified set of vertices. In ERM, each US denominated cash flow is mapped to one or more of the vertices shown below:

$$\leq 1\,\text{yr} \quad 2\,\text{yrs} \quad 3\,\text{yrs} \quad 5\,\text{yrs} \quad 7\,\text{yrs} \quad 10\,\text{yrs} \quad 15\,\text{yrs} \quad 20\,\text{yrs} \quad \geq 30\,\text{yrs} \qquad (3.2.1)$$

Mapping a cash flow means splitting it between two adjacent vertices in such a way that both the present value of the cash flow and its sensitivity to the zero rates are preserved. As a result of mapping, a portfolio of instruments is transformed into a portfolio of standard cash flows. Figure 1 shows how the actual cash flow at year six is split into the synthetic cash flows at the five-year and seven-year vertices.

RiskMetrics documentation ([5], pp. 43–45) shows that a payment of USD 1 at time t could be

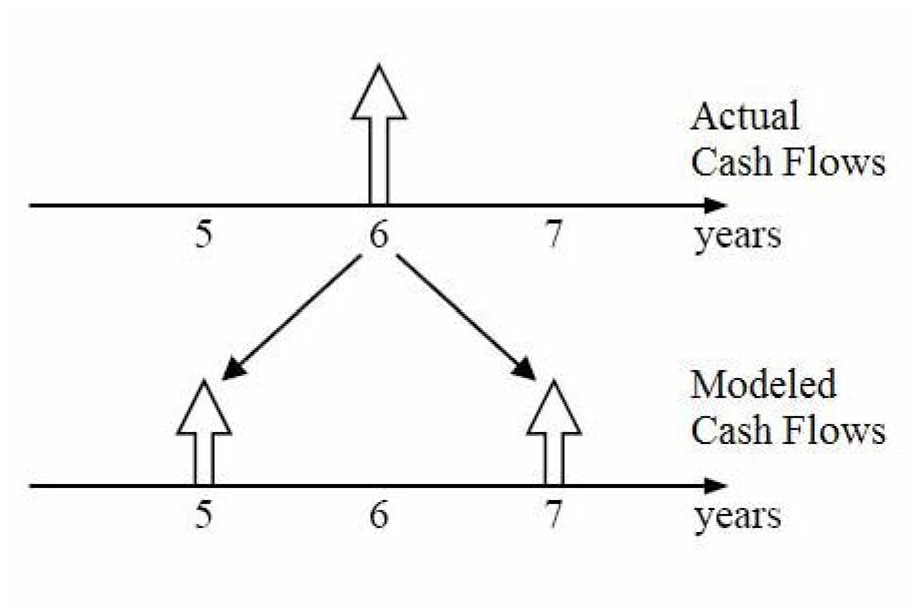

Figure 1. Cash Flow Mapping

mapped into a payment of W_L at time t_L, a payment of W_R at time t_R (t_L and t_R are the two adjacent vertices around t, $t_L < t < t_R$), and a cash position C:

$$W_L = \alpha \frac{t}{t_L} e^{-z_t t} e^{z_L t_L}$$

$$W_R = (1-\alpha) \frac{t}{t_L} e^{-z_t t} e^{z_R t_R} \qquad (3.2.2)$$

$$C = -\frac{(t - t_L)(t_R - t)}{t_R t_L} e^{-z_t t}$$

This rule assumes that the zero rate z_t for maturity t is calculated as a linear interpolation of zero rates z_L and z_R at the vertices, i.e.,

$$z_t = \alpha z_L + (1-\alpha) z_R, \qquad (3.2.3)$$

where $\alpha = (t_R - t)/(t_R - t_L)$.

3.2.1.2. Financial Risk Factors

As discussed above the Seabury enterprise risk management platform relies on a multi-factors methodology employed by RiskMetrics risk model. RiskMetrics does not look at the company portfolio as a set of assets which ought to be processed independently, but analyzes it in terms of common risk factors affecting the value of assets. Seabury ERM utilizes such risk factors as interest rates of different maturities for the U.S. dollar (USD) and other currencies, equity sector returns for the United States and other countries, and foreign exchange rates. This multi-factor approach is widely accepted in the financial industry due to its practicality. One of the benefits of this approach is that the assessment of risk that is incorporated into a complex portfolio structure could be reduced to the analysis of a limited number of risk factors; correlations between different asset classes and risk categories will be derived straightforwardly through the exposures to the specific risk factors. Another benefit is that, through the use of a comprehensive set of risk factors, the analyst may model various market environments and evaluate the impact to the company arising from the change in market conditions.

All factors employed by ERM can be observed directly in the market, therefore important factor characteristics such as volatilities and correlations of the returns can be obtained directly from the historical price series through the methods of statistical analysis. The distribution of past returns can then be modeled to provide a reasonable forecast of future factor returns over the required horizon. For each individual instrument, Seabury ERM identifies the set of the specific risk factors which drive the change in the instrument price as well as the exposure to each factor. By generating future scenarios for each risk factor, ERM infers changes in the instrument value and re-prices the total company portfolio accordingly. Such a bottom-up approach possesses a great degree of flexibility and simplifies the broad analysis of the company.

We follow the methodology of the RiskMetrics deriving distributions of parameters for risk factors from the historical logarithmic returns series:

$$r_{t,H} = \ln\left(P_{t+H} / P_t\right), \qquad (3.2.4)$$

where $r_{t,H}$ denotes the return from time t over the horizon period H to $t + H$ and P is a generalized price which, depending on the risk factor, may represent the Treasury bond price, industry index value, or exchange rate. RiskMetrics advocates the use of the exponentially weighted returns for the estimation of volatility; this schema assigns more weight to the most recent data and limits the effective number of historical returns. While this approach seems appropriate for a short-term horizon, it is not suitable for a one-year horizon which is considered necessary due to the nature of insurance liabilities. Taking these issues into account the authors' selected the equally weighted volatility estimate, which uses historical data series going back 10 years.

The model for the distribution of future returns is based on the notion that logarithmic returns of risk factors are jointly normally distributed. J. Mina and J. Yi Xiao [5] outline the arguments that justify the practical use of normal distributions for the problem at hand—fast and accurate estimation of various risk statistics for a portfolio driven by a large number of risk factors.

A practical justification for the normal distribution is the simplicity of its calibration. The univariate normal distribution can be described by two parameters that are easy to calibrate: the mean and standard deviation. Every distributional model has to consider the dependence structure of the returns as well as their stand-alone characteristics. The most important practical advantage of the multivariate normal distribution is that its dependence structure is uniquely defined by a correlation matrix.

Normal distributions though can not adequately describe rare events which result in big losses, such as a catastrophe loss. Modeling such an event would require a skewed distribution with a heavy negative tail. This issue will be discussed in more details in the CAT risk section.

It is essential that all generalized prices used for the returns and subsequent correlation estimates must be denominated in USD. Use of a single currency across all the instruments either domestic or foreign in order to produce a correlation structure simplifies a transition from one reporting currency to another (rebasing) with no recalculations of the correlation matrix and principal components required.

To illustrate this concept let us consider a return on 10-year Treasury strip. The price of the instrument is denoted as $P^{10\,\mathrm{y}}$. As long as the base currency is USD, the return is a relative change in the domestic price:

$$r_{t,H}^{10\,\mathrm{y}\,USD} = \ln\left(F_{t+H}^{10\,\mathrm{y}\,USD} / P_{t}^{10\,\mathrm{y}\,USD} \right) \tag{3.2.5}$$

If the company operates in Europe and reports its earnings and risks in EUR, then from its perspective, a holding of the US Treasury instrument in the investment portfolio would be a subject to EUR/USD exchange rate risk. Even if domestic price of the treasury stays the same over the horizon period, a change in EUR/USD rate can increase or reduce its value for a foreign company:

$$
\begin{aligned}
r_{t,H}^{10\,\mathrm{y}\,EUR} &= \ln\left(P_{t+H}^{10\,\mathrm{y}\,EUR} / P_{t}^{10\,\mathrm{y}\,EUR} \right) \\
&= \ln\left(P_{t+H}^{10\,\mathrm{y}\,\mathrm{USD}} \cdot [USD/EUR]_{t+H} / P_{t}^{10\,\mathrm{y}\,\mathrm{USD}} \cdot [USD/EUR]_{t} \right) \\
&= \ln\left(P_{t+H}^{10\,\mathrm{y}\,\mathrm{USD}} / P_{t}^{10\,y\,\mathrm{USD}} \right) + \ln\left([USD/EUR]_{t+H} / [USD/EUR]_{t} \right) \\
&= r_{t,H}^{10\,\mathrm{y}\,\mathrm{USD}} - r_{t,H}^{[EUR/USD]}
\end{aligned}
\qquad (3.2.6)
$$

In other words, the Euro-denominated return on the Treasury could be derived from the USD-denominated return by simply subtracting the return on the EUR/USD exchange rate. Since the same is true for any USD-denominated instrument, the rebasing schema could be depicted as follows:

1. Simulate USD-denominated returns for all risk factors
2. Simulate CCY/USD returns for a new reporting currency CCY for each scenario
3. Recalculate factor returns by applying simulated foreign exchange rates
4. Apply new factor returns and produce a new company's value for each scenario. Calculate new CCY-denominated risk statistics from the new distribution of the company values.

3.2.1.3. Risk Categories

The bottom-up approach allows for the analysis of the various aspects of a company's risk. Breaking risk down into its subcategories proved very useful for understanding the uncertainties faced by the company and protecting it from potential losses. These subcategories are nothing but different views of the same risk. ERM emphasizes the following categories:

Insurance Risk—the uncertainty associated with future payments of insurance liabilities. The factors driving this risk are the size and structure of the insurance business. ERM measures this risk from the analysis of the company's loss triangles and historical underwriting results.

Credit risk—the uncertainty associated with changes in obligor credit quality. On the investment side, this category indicates the potential loss in the net worth the company may experience from the deterioration in the credit quality of its investments assets. On the insurance side, the main component of the credit risk is the credibility of the reinsurers who may fail to pay on their obligations. ERM measures credit risk from the historical rating upgrade and downgrade records.

Interest rate risk—the uncertainty associated with a change in interest rates. It measures a change in the net value of fixed income instruments and insurance liabilities resulting from the potential fluctuations in interest rates. In most cases, variations in future interest rates impact present value through the adjustment of discounting factors. But for some instruments like callable bonds or ABS/MBS, changing interest rates may impact the projected cash flows. ERM estimates interest rate risk parameters from the historical variations in government yield curves.

Equity Risk—the uncertainty associated with the stock market volatility. ERM applies the historical experience of stock market movements to the investment portfolio in order to assess a potential loss in equity positions.

Currency Risk—the uncertainty associated with fluctuations in exchange rates. Measures potential loss for the company which is doing business abroad and/or keeps instruments denominated in foreign currency in its investment portfolio. ERM estimates this risk from the historical variations in foreign exchange rates.

Catastrophe Risk—the uncertainty associated with the impact that natural catastrophes may have on a value of the company. We will discuss this risk in the Catastrophe Risk subsection of Section 3.2.1.5.

It is important to note that risk subcategories are not independent. Even though each particular category represents a distinct aspect of the enterprise risk, they are driven by the common set of factors which, in turn, are closely correlated to each other. Exchange rates are not independent from the interest rates of the participating currencies and the credit rating of a company may be closely related to its stock performance. As a result, the total risk of the company may be significantly lower than the sum of the individual risks. The difference between the two indicates the level of correlation that exists between risk categories and is usually referred to as the diversification benefit. Unlike many risk management systems that rely on a top-down approach, ERM does not make any assumptions about correlations between risk categories and the resulting risk reduction. Estimates of correlation arise logically from the bottom-up analyses. We will discuss this issue in more detail in the context of risk aggregation capabilities offered by ERM.

3.2.1.4. Investment Portfolio

Common Stock

Seabury ERM employs the linear regression model assuming that the standardized log return of the firm's value, r^e, is the weighted average of two standardized returns, namely, the industry return, r_I, and the firm-specific return, ε:

$$r^e = w_I r_I + \sigma \cdot \sqrt{1 - w_I^2}\varepsilon \qquad (3.2.7)$$

where $\varepsilon \sim N(0,1)$ and volatility σ could be derived from the historical stock prices

The practical interpretation of the above equation is that the firm's return can be sufficiently explained by the index return of the industry classification to which the firm belongs, with a residual part that can be explained solely by information unique and specific to the firm. Firm-specific risk can generally be considered to be a function of company asset size. Larger companies tend to have smaller firm-specific risk while smaller companies, on the other hand, tend to have larger firm-specific risk. According to JP Morgan's CreditManager, the firm-specific risk follows the logistic curve:

$$FirmSpecificRisk = \frac{1}{2\left(1 + Assets^{0.4884} \times e^{-12.4739}\right)}, \qquad (3.2.8)$$

with *Assets* being the total assets in USD. For asset size of $1 billion, firm-specific risk is 0.46, implying $w_I = 0.54$. For asset size of $100 billion, $w_I = 0.75$. Each simulation scenario produces a realization for all index returns and specific returns for all stocks positions, thus assigning new value for the equity portfolio.

Risk-free bonds

ERM views a risk-free coupon-paying bond as a deterministic stream of future cash flows. Applying cash-flow mapping procedure as described in Section 3.2.1.1 above, ERM maps the future payments into individual vertices denoted as W_{t_i}. To calculate horizon value of the bond, a cash flow at every vertex is discounted using the appropriate domestic risk-free curve. The authors selected to use U.S. and foreign synthetic zero curves provided by Bloomberg® for discounting cash flows. This procedure can be repeated for all coupon-paying bonds held in the company's portfolio. Cash flows from individual instruments are aggregated into the suitable maturity vertices.

The market value of the bond portfolio becomes

$$V_h = \sum_{t_i \in \text{vertices}} \left(\sum_j W_{t_i}^j \right) e^{-z_{t_i} t_i} + \sum_j C^j \,, \tag{3.2.9}$$

where index j denotes the individual bond; z_{t_i} is the zero rate with the maturity t_i, and C is a cash position produced from the mapping algorithm. For each scenario, ERM generates the array of simulated zero rates, substitutes them into the equation, and calculates a new portfolio value. Simulations therefore result in a distribution of the projected portfolio values.

For cash flows that are within the time horizon, ERM takes the conservative approach and assumes that the cash flow earns the interest at the constant risk-free short-term rate and so the present value at the horizon is just the sum of the accrued cash flows.

Risky bonds

Unlike risk-free bonds, where future cash flows are deterministic and the projected market value is only subject to interest rate uncertainty, the risky bonds have exposure to default risk as well. To capture this risk, ERM models the change in the credit quality of the bond over the specified horizon through the use of a transition probability matrix—rules that shows how credit ratings migrate over unit time intervals. Whether the credit rating of the bond improves, deteriorates, or stays the same, the market value of the instrument adjusts accordingly. Table 1 below shows transition probabilities and resulting values of a hypothetical BBB bond over a one-year period.

Table 1. Year-End Values after Credit Rating Migration from BBB

Current Rating	Possible Future Rating	Probability	Resulting Value
	AAA	0.02%	$101.69
	AA	0.33%	$101.47
	A	5.95%	$101.03
BBB	BBB	86.9%	$100.00
	BB	5.30%	$94.86
	B	1.17%	$91.21

	C	0.12%	$77.77
	D	0.18%	$47.54

ERM employs the CreditMetrics' asset value model which links the return on a company's stock with its probability of being upgraded or downgraded within the examined period of time. The asset value model assumes that the one-year return is normally distributed and that the bond's rating changes to a new value when the normalized return drops below or jumps above the respective threshold as illustrated by the following chart. The thresholds could be calculated from transition probabilities as dictated by a normal distribution:

$$\begin{aligned} \text{Prob}(\mathit{Default}) &= \text{Prob}(r < Z_D) = \Phi(Z_D) \\ \text{Prob}(\mathit{CCC}) &= \text{Prob}(Z_{CCC} < r < Z_D) = \Phi(Z_C) - \Phi(Z_D), \end{aligned} \quad (3.2.10)$$

which yields for the threshold S:

$$Z_S = \Phi^{-1}\left[\sum_{l=D}^{S} \text{Prob}(l)\right], \qquad S = CCC, B, \cdots, AAA. \quad (3.2.11)$$

ERM thus evaluates the credit risk embedded in a corporate bond by simulating a return on the issuing firm's stock price. For a portfolio of risky bonds, the co-movements in credit migrations of different bonds are captured through the simulation of correlated returns of the corresponding stock prices.

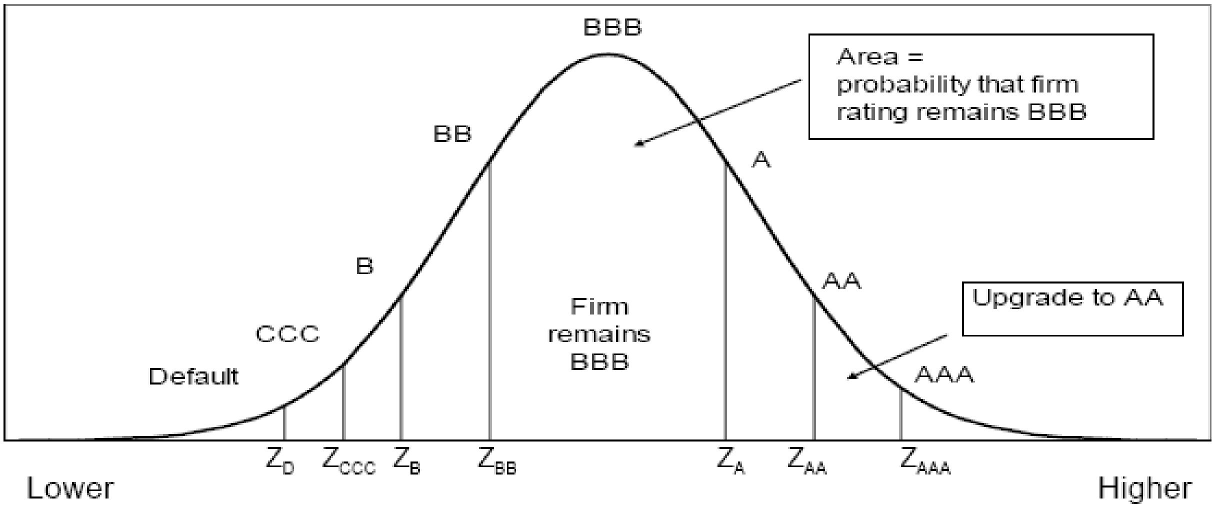

Figure 2. Distribution of Asset Returns with Rating Change Thresholds

Interest rate risk assessment for a risky bond is analogous to that for a risk-free bond explained in the previous section.

Many sovereign and corporate bonds carry a call provision which grants the issuer an option to retire (or “call”) the bond prior to its maturity. The callable bond value therefore equals the “optionless” bond value, less the call option value. Since the option value depends primarily on the current interest rates, and changes along with the changes in a yield curve, this value should be recalculated for each scenario whenever interest rates fluctuate. ERM utilizes the Hull-White model of interest rate evolution to calculate a value of the callable bond for each simulation scenario.

ABS/MBS

When calculating risk for ABS/MBS securities, one must take into consideration that these instruments (unlike bonds) carry a prepayment provision granted to the borrower. This means ABS/MBS may be fully or partially prepaid by the borrower at any time he or she selects. This option has a significant importance; its valuation becomes the integral part of ABS/MBS full valuation. The proper option valuation requires the use of a prepayment model, which utilizes a wide range of historical data and analyzes different economic factors. Implementing such a model would go beyond the scope of the ERM platform, instead, ERM relies on key rate durations and the expected horizon prices provided by the user or calculated with specialized software like that developed by CMS BondEdge® or Citigroup YieldBook®, or other vendors. Key rate durations (also called option-adjusted durations) reflect the change in instrument value with respect to a small change in an interest rate for a specific maturity bucket (key rate). Part of this value change comes from the variation in prepayment speed, driven mainly by interest rates; therefore this method captures prepayment risk rooted into the total instrument risk. ERM calculations also include convexity which guarantees a second order of accuracy. Effective duration and convexity are estimated from the parallel shift of the entire yield curve.

For *i*th scenario change in ABS/MBS market value at the horizon becomes

$$MV_h^i = MV_h^0 \cdot \left(1 - EDUR \cdot \overline{\Delta z^i} + \frac{1}{2} ECNVX \cdot \overline{\Delta z^i}^2 - \sum_{j=1}^{N} KRD_j \cdot \left(\Delta z_j^i - \overline{\Delta z^i}\right)\right), \quad (3.2.12)$$

where

$$\overline{\Delta z^i} = \frac{1}{N} \sum_{j=1}^{N} \Delta z_j^i , \quad (3.2.13)$$

is an averaged parallel shift, Δz_j^i is a fluctuation of a zero key rate of *j*th maturity bucket around its central horizon value for *i*th scenario. In Eq. (3.2.12), the effective duration *EDUR* and convexity *ECNVX* are the first and second degree order changes in the ABS/MBS price with respect to a small parallel shift in the yield curve. Like *KRD*s, they encompass the impact that the changing interest rates may have on prepayment speed.

Preferred stock

Usually insurance companies hold only a small portion of the investment portfolios (up to 2%) in preferred stocks. More than 90% of them are of callable non-convertible cumulative types which are similar to corporate bonds. To asses their risk ERM treats preferred stock as callable corporate bonds with the identical maturity, face value, coupon percentage, and rating. If stock does not have a maturity, ERM assigns it the longest maturity term available for a given currency. Given that preferred stocks are subordinate to even the least senior bonds, ERM sets their recovery rate to zero.